

Name Key

Date \_\_\_\_\_

Period \_\_\_\_\_

## Writing Biconditional Statements // Truth Tables

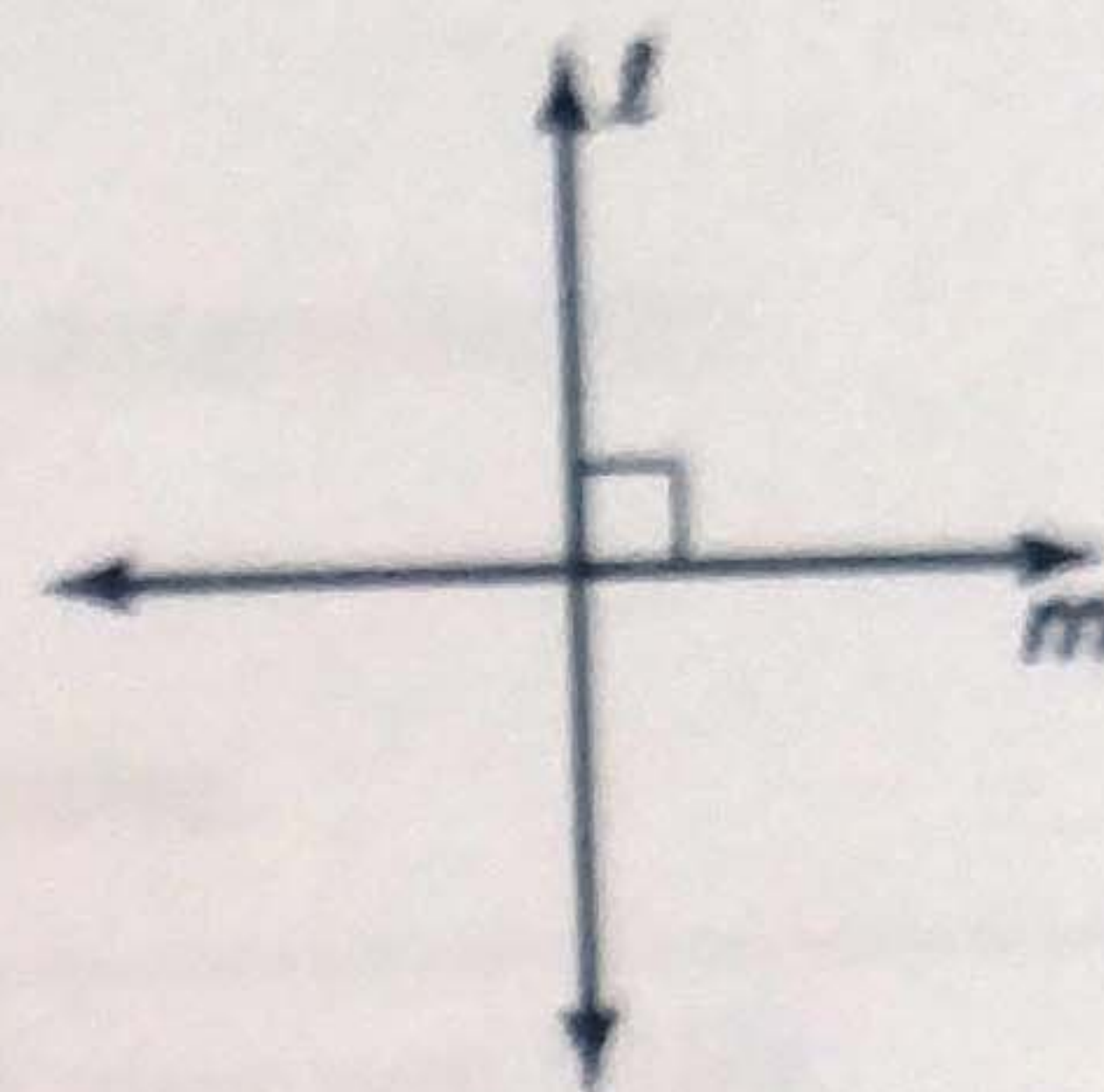
## Lesson Objective

WRITE BICONDITIONAL STATEMENTS; WRITE DEFINITIONS AS BICONDITIONAL STATEMENTS; FILL OUT TRUTH TABLES TO FIND LOGICALLY EQUIVALENT STATEMENTS.

## Using Definitions

You can write a definition as a conditional statement in "if-then" form or as its converse. Both the conditional statement AND converse are true for definitions. For example, consider the following definition of *perpendicular lines*:

If <sup>"p"</sup> two lines intersect to form a right angle, then <sup>"q"</sup> they are perpendicular lines.



Now, write the converse of the definition:

IF TWO LINES ARE PERPENDICULAR, THEN THEY INTERSECT TO FORM A RIGHT ANGLE.

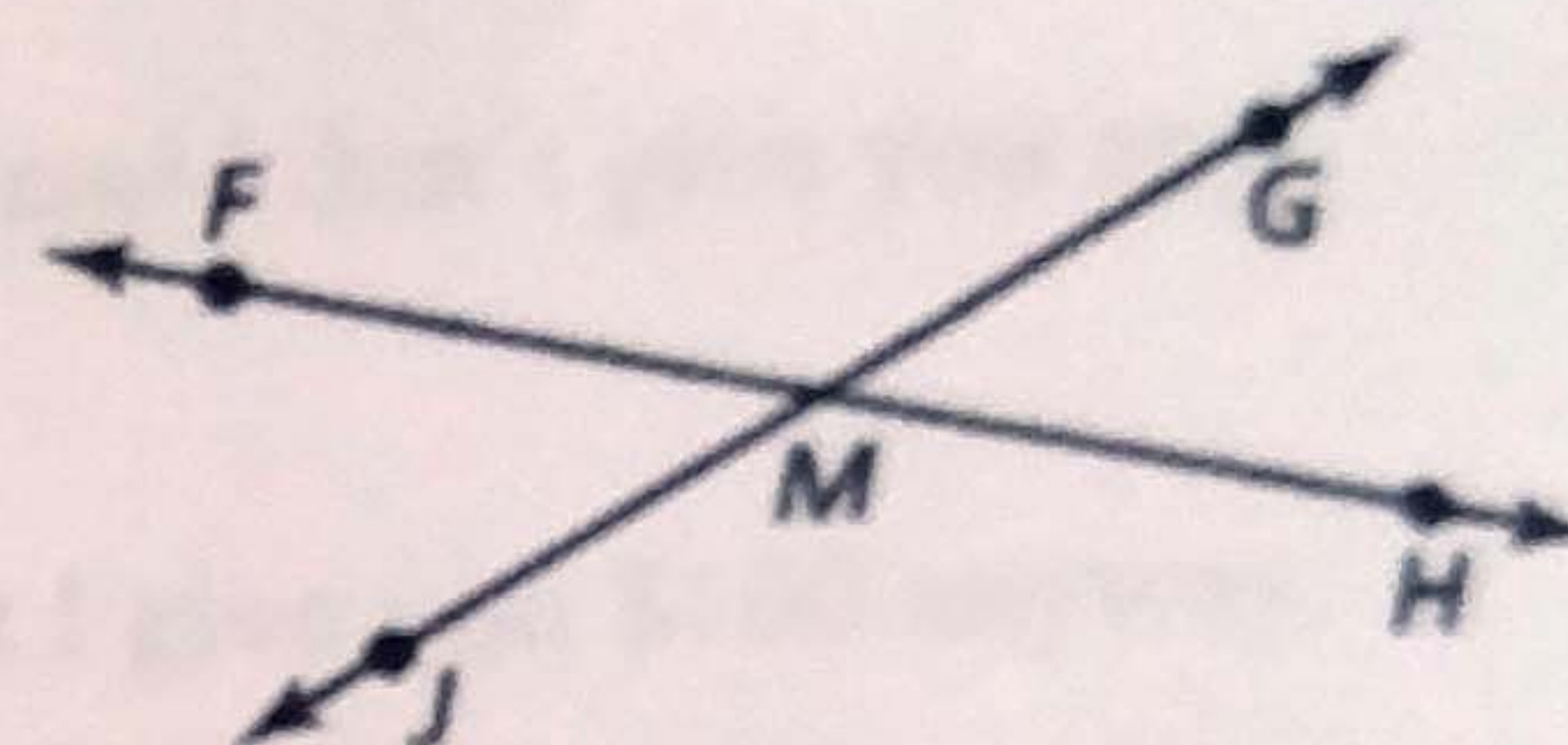
Example: Use the diagram. Decide whether each statement is true. Explain your answer using the definitions you have learned from Chapter 1 and above.

1.  $\angle JMF$  and  $\angle FMG$  are supplementary.

2. Point M is the midpoint of  $\overline{FH}$ .

3.  $\angle JMF$  and  $\angle HMG$  are vertical angles.

4.  $\overline{FH} \perp \overline{JG}$



Biconditional Statements: When a conditional statement and its converse are both TRUE, you can write them as a single biconditional statement.

→ A biconditional statement is a statement that contains the phrase "IF AND ONLY IF (iff)".

Words: "p" if and only if "q"

Symbols:  $p \leftrightarrow q$

\*\*We saw above that definitions have true conditional and converse statements. So, ANY definition can be written as biconditional statements!

## Example:

5. Rewrite the definition of perpendicular lines above as a single biconditional statement.

TWO LINES INTERSECT TO FORM A RIGHT ANGLE iff "IF AND ONLY IF"  
THEY ARE PERPENDICULAR LINES.



6. Rewrite the definition of congruent segments (shown below) as a single biconditional statement:  
 "If two line segments have the same length, then they are congruent segments."

TWO LINE SEGMENTS HAVE THE SAME LENGTH iff  
 THEY ARE CONGRUENT SEGMENTS

7. Write the conditional statement and converse within the biconditional.

"An angle is a right angle if and only if its measure is  $90^\circ$ ."

Conditional: IF AN ANGLE IS A RIGHT ANGLE, THEN ITS MEASURE IS  $90^\circ$ .

Converse: IF AN ANGLE HAS A MEASURE OF  $90^\circ$ , THEN IT IS A RIGHT ANGLE.

### Truth Tables

**Truth Value:** The truth value of a statement is either true ( T ) or false ( F ). You can determine the conditions under which a conditional statement is true by using a **truth table**.

Let's look at conditional statements bring TRUE or FALSE in the context of keeping a promise.

Conditional		
$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

**MY PROMISE:** If there is an elephant in the library, then I will give you \$100.

1. Suppose there really IS an elephant in the library, and I give you \$100.
2. Suppose there really IS an elephant in the library, and I don't give you any money.
3. Suppose there is not an elephant in the library, but I give you \$100 anyway. (SWEET!)
4. Suppose there is not an elephant in the library, and I don't give you any money.

Now, let's fill out the following truth tables for the converse, inverse, and contrapositive of a conditional statement:

Converse		
$p$	$q$	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse				
$p$	$q$	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contrapositive				
$p$	$q$	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Think About It... Two statements are LOGICALLY EQUIVALENT when they have the same truth table.

So, which statements define the term above? (HINT: look at their truth table outcomes!)

CONDITIONAL and CONTRAPOSITIVE  
CONVERSE and INVERSE