

3.5 Notes (Day 2): Finding Equations of Parallel and Perpendicular Lines

Slope-Intercept Form:

Here is the usual format for the questions you will see:

- Find the equation of the line that is *parallel* to the line $3x - 5y = 9$ and through $(-6, 5)$.

Step 1: I need to do is solve " $3x - 5y = 9$ " for " $y =$ ", so that I can find my reference slope:

$$3x - 5y = 9 \quad \text{Subtract } 3x \text{ from both sides}$$

$$-5y = -3 + 9 \quad \text{Divide by } -5 \text{ on both sides}$$

$$y = \left(\frac{3}{5}\right)x - \left(\frac{9}{5}\right)$$

So the reference slope from the reference line is $m = \frac{3}{5}$.

Step 2: Since a parallel line has an identical slope, then the parallel line through $(-6, 5)$ will have slope $m = \frac{3}{5}$. Hey, now I have a point and a slope!

Note: The y-intercept for the line $3x - 5y = 9$ is $-\frac{9}{5}$. The y-intercept has nothing to do with the final answer to this type of question and can be ignored in every problem. All you need when writing the new equation is the slope!

Step 3: Now let's plug the slope and the given point into the equation and solve for b :

$$y = mx + b \quad \text{Plug in the values: } \frac{3}{5} \text{ is } m(\text{the slope}), -6 \text{ is } x, 5 \text{ is } y$$

$$5 = \left(\frac{3}{5}\right)(-6) + b \quad \text{Multiply } \left(\frac{3}{5}\right)(-6)$$

$$5 = \left(-\frac{18}{5}\right) + b \quad \text{Multiply both sides by 5 to get rid of the denominator}$$

$$25 = -18 + 5b \quad \text{Add 18 to both sides}$$

$$43 = 5b \quad \text{Divide by 5 on both sides}$$

$$\frac{43}{5} = b$$

Step 4: Now let's write the equation of a line using the slope and y-intercept we just found.

$$y = \left(\frac{3}{5}\right)x + \left(\frac{43}{5}\right) \quad \text{All done!}$$

- Find the equation of the line that is *perpendicular* to the line $8x + 6y = 15$ and through $(3, -4)$.

Step 1: I need to do is solve " $8x + 6y = 15$ " for " $y =$ ", so that I can find my reference slope:

$$8x + 6y = 15 \quad \text{Subtract } 8x \text{ from both sides}$$

$$6y = -8x + 15 \quad \text{Divide by 6 on both sides}$$

$$y = \left(-\frac{4}{3}\right)x + \left(\frac{5}{2}\right)$$

So the reference slope from the reference line is $m = -\frac{4}{3}$.

Step 2: For a perpendicular line, I have to find the perpendicular slope. The reference slope is $m = -\frac{4}{3}$, and, for the perpendicular slope, I'll flip this slope and change the sign. Then the perpendicular slope is $m = \frac{3}{4}$.

Step 3: Now let's plug the perpendicular slope and the given point into the equation and solve for b :

$$y = mx + b \quad \text{Plug in the values: } \frac{3}{4} \text{ is } m(\text{the } \perp \text{ slope}), 3 \text{ is } x, -4 \text{ is } y$$

$$-4 = \left(\frac{3}{4}\right)(3) + b \quad \text{Multiply } \left(\frac{3}{4}\right)(3)$$

$$-4 = \left(\frac{9}{4}\right) + b \quad \text{Multiply both sides by 4 to get rid of the denominator}$$

$$-16 = 9 + 4b \quad \text{Subtract 9 on both sides}$$

$$-25 = 4b \quad \text{Divide by 4 on both sides}$$

$$-\frac{25}{4} = b$$

Step 4: Now let's write the equation of a line using the slope and y-intercept we just found.

$$y = \left(\frac{3}{4}\right)x - \left(\frac{25}{4}\right) \quad \text{All done!}$$

Now solve and graph examples 1 and 2 on your notes worksheet.



Point-Slope Form:

Here is the usual format for the questions you will see:

- **Find the equation of the line that is *parallel* to the line $2x - 3y = 9$ and through $(4, -1)$.**

Step 1: I need to do is solve " $2x - 3y = 9$ " for " $y =$ ", so that I can find my reference slope:

$$2x - 3y = 9 \quad \text{Subtract } 2x \text{ from both sides}$$

$$-3y = -2x + 9 \quad \text{Divide by } -3 \text{ on both sides}$$

$$y = \left(\frac{2}{3}\right)x - 3$$

So the reference slope from the reference line is $m = \frac{2}{3}$.

Step 2: Since a parallel line has an identical slope, then the parallel line through $(4, -1)$ will have slope $m = \frac{2}{3}$. Hey, now I have a point and a slope!

Step 3: So I'll use the point-slope form to find the line:

$$y - (-1) = \left(\frac{2}{3}\right)(x - 4) \quad \text{Plug in the values: } \frac{2}{3} \text{ is } m(\text{the slope}), 4 \text{ is } x, -1 \text{ is } y$$

$$y + 1 = \left(\frac{2}{3}\right)x - \frac{8}{3} \quad \text{Distribute the } \frac{2}{3}$$

$$y = \left(\frac{2}{3}\right)x - \frac{8}{3} - \frac{3}{3} \quad \text{Simplify } -\frac{8}{3} - \frac{3}{3}$$

$$y = \left(\frac{2}{3}\right)x - \left(\frac{11}{3}\right) \quad \text{All done!}$$

- **Find the equation of the line that is *perpendicular* to the line $2x - 3y = 9$ and through $(4, -1)$.**

Step 1: I need to do is solve " $2x - 3y = 9$ " for " $y =$ ", so that I can find my reference slope:

$$2x - 3y = 9 \quad \text{Subtract } 2x \text{ from both sides}$$

$$-3y = -2x + 9 \quad \text{Divide by } -3 \text{ on both sides}$$

$$y = \left(\frac{2}{3}\right)x - 3$$

So the reference slope from the reference line is $m = \frac{2}{3}$.

Step 2: For a perpendicular line, I have to find the perpendicular slope. The reference slope is $m = \frac{2}{3}$, and, for the perpendicular slope, I'll flip this slope and change the sign. Then the perpendicular slope is $m = -\frac{3}{2}$.

Step 3: Now I can use the point-slope form to find the line. Note that the only change from the calculations I just did is that the slope is different now.

$$y - (-1) = \left(-\frac{3}{2}\right)(x - 4) \quad \text{Plug in the values: } -\frac{3}{2} \text{ is } m(\text{the } \perp \text{ slope}), 4 \text{ is } x, -1 \text{ is } y$$

$$y + 1 = \left(-\frac{3}{2}\right)x + 6 \quad \text{Distribute the } -\frac{3}{2}$$

$$y = \left(-\frac{3}{2}\right)x + 5 \quad \text{All done!}$$

Now solve and graph examples 3 and 4 on your notes worksheet.

Note: If a question asks you whether two given lines are "parallel, perpendicular, or neither", you must answer that question by finding their slopes, *not by drawing a picture!* Pictures can only give you a rough idea of what is going on. Find the slopes using the slope formula; don't just draw the pictures!

Hope this helps! Sorry I'm out today ☹ Check your answers to the notes worksheet by looking on my website, and be ready with any questions for the Chapter 3 Review tomorrow. ~MRS. GARNET

*****Reminder: Chapter 3 Test will be Thursday, November 3. Yes, THIS Thursday.*****