Name KEY Date ____________ Period ____________

Inductive Reasoning // Counterexamples

Lesson Objective

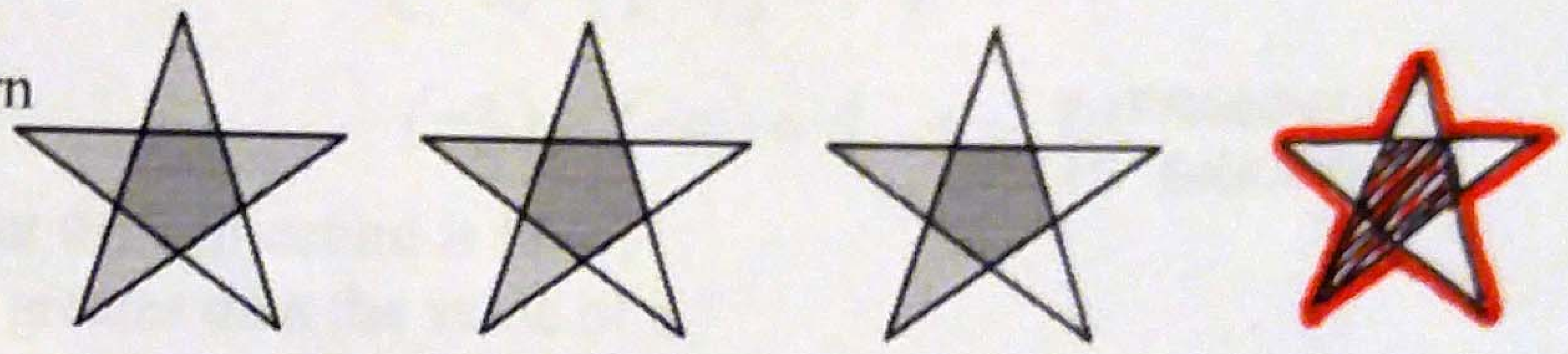
MAKE AND TEST CONJECTURES USING INDUCTIVE REASONING; FIND COUNTEREXAMPLES TO PROVE STATEMENTS FALSE.

Conjecture: A conjecture is an unproven statement based on observations.

Inductive Reasoning: when you find a PATTERN in specific cases and then write a conjecture for the general case.

Example:

1. Sketch the next figure in the pattern



2. Sketch the next figure in the pattern

Making and Testing a Conjecture

Example:

3. Numbers such as 3, 4, and 5 are called CONSECUTIVE INTEGERS. Make a test a conjecture about the sum of any three consecutive integers.

$1 + 2 + 3 = 6 = 2 \cdot 3$

$7 + 8 + 9 = 24 = 8 \cdot 3$

$20 + 21 + 22 = 63 = 21 \cdot 3$

CONJECTURE:

SUM OF THREE CONSECUTIVE INTEGERS EQUALS 3 TIMES THE MIDDLE NUMBER

TEST:

$199 + 200 + 201 \stackrel{?}{=} 200 \cdot 3$

$600 = 600$ ✓

~~4.~~ Make and test a conjecture about the product of two odd integers.

SKIP

Counterexamples

To show a conjecture is true, you must show that it is true for ALL cases.

You can show it is false, however, by finding just ONE COUNTEREXAMPLE.

A counterexample is a specific case for which the conjecture is false.

Example:

5. A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

   "The sum of two numbers is always greater than their difference."

   WORKS:

   $2 + 3 = 5$ ← BIGGER

   $2 - 3 = -1$

   COUNTEREXAMPLE:

   $(-3) + (-4) = -7$

   $(-3) - (-4) = 1$ ← DIFFERENCE IS GREATER!

6. Find a counterexample to show that the conjecture is false.

   "The value of x^2 is always greater than the value of x."

   WORKS:

   $3^2 = 9$

   $9 > 3$ ✓

   COUNTEREXAMPLE:

   $(\frac{1}{2})^2 = \frac{1}{4}$

   $\frac{1}{4} \ngtr \frac{1}{2}$ ✗

7. Find a counterexample to show that the conjecture is false.

   "If $1 - y > 0$, then $0 < y < 1$."

   WORKS:

   $1 - \frac{1}{2} = \frac{1}{2}$

   $\frac{1}{2} > 0$ ✓

   $0 < \frac{1}{2} < 1$ ✓

   COUNTEREXAMPLE:

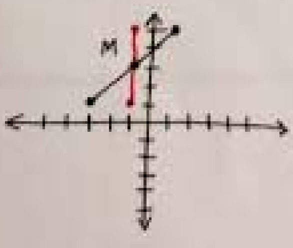
   $1 - (-3) = 4$

   $4 > 0$ ✓

   $0 \nless -3 \nless 1$ ✗

8. Determine if each biconditional is true. If false, provide a counterexample.

   "A segment has endpoints at (1, 5) and (-3, 1) if and only if its midpoint is at (-1, 3)."



   COUNTEREXAMPLE:

   ENDPOINTS (-1, 1) AND (-1, 5) ALSO HAS MIDPOINT (-1, 3).